What function do each of the following represent?

(a) $\sum_{n=0}^{\infty} \frac{x^{n+2}}{n!}$

(b) $\sum_{n=2}^{\infty} x^n$

(c) $\sum_{n=0}^{\infty} \left(\frac{x^n}{n!} + x^n\right)$

(d) $\sum_{n=-1}^{\infty} x^{n+1}$

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(a) $\sum_{n=0}^{\infty} \frac{x^{n+2}}{n!}$

$= x^2 + x^3 + \frac{x^4}{2!} + \frac{x^5}{3!} + \frac{x^6}{4!} + \cdots$

$= x^2\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots\right)$

$= x^2 e^x$

(b) $\sum_{n=2}^{\infty} x^n$

$= x^2 + x^3 + x^4 + \cdots$

$= x^2(1 + x + x^2 + \cdots)$

$= x^2 \sum_{n=0}^{\infty} x^n$

$= x^2\left(\frac{1}{1-x}\right)$

$= \frac{x^2}{1-x} \quad (|x|<1)$

(c) $\sum_{n=0}^{\infty} \left(\frac{x^n}{n!} + x^n\right)$

$= \sum_{n=0}^{\infty} \frac{x^n}{n!} + \sum_{n=0}^{\infty} x_n$

$= e^x + \frac{1}{1-x}$

(d) $\sum_{n=-1}^{\infty} x^{n+1}$

$= \sum_{n=0}^{\infty} x^n$

$= \frac{1}{1-x} \quad (|x|<1)$